GEOMETRY ON THE PLANE (1)

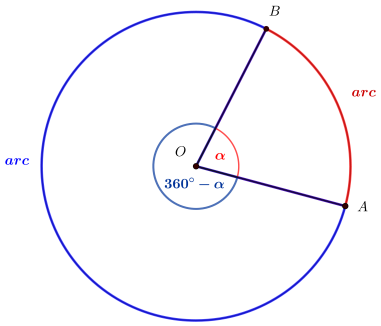
ARC, SECTOR,ANGLES IN A CIRCLE

VOCABULARY & PROPERTIES

I. DEFINITIONS

1. **Arc**. An arc is a portion of a circle defined by two endpoints. A **central angle** separates the circle into two arcs with measures related to the measure of the central angle. The lenght of an arc is equal to

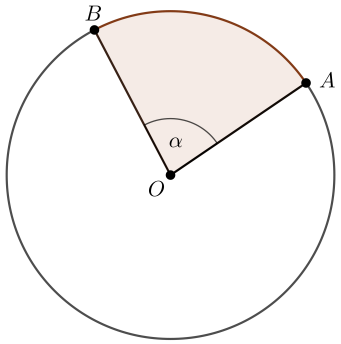
$$L = \frac{\alpha \cdot 2\pi \cdot r}{360^\circ}$$



2. **Sector**. A sector of a circle, with center O, is the region bounded by an arc AB of the circle and two radii OA and OB. The area of a sector is equal to

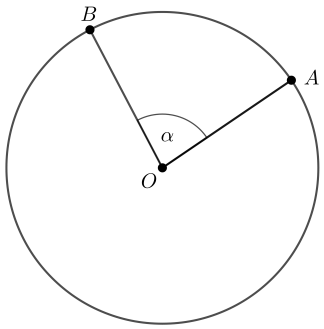
$$A = \frac{\alpha \cdot \pi \cdot r^2}{360^\circ},$$

where r is the radius and α is the angle on radians.

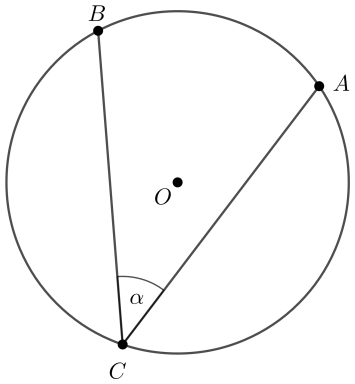


3. **Central angle** An angle whose vertex is in the center of a given circle and whose legs (sides) are radii intersecting the circle in two distinct points A and B. Central angles are subtended by an arc between those two points, and the arc length is the central angle of a circle of radius one (measured in radians). The central angle is also known as the **arc's angular distance**. The size of a central angle α is $0^{circ} < \alpha < 360^\circ$ or $0 < \alpha < 2\pi$ (radians). When defining or drawing a central angle, in addition to specifying the points A and B, one must specify

whether the angle being defined is the convex angle ($< 180°$) or the reflex angle ($> 180°$). Equivalently, one must specify whether the movement from point A to point B is clockwise or counterclockwise.

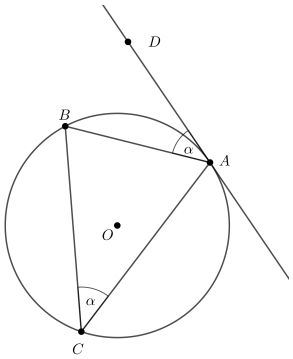


4. **An inscribed angle** is an angle whose vertex sits on the circumference of a circle. The vertex is the common endpoint of the two sides of the angle. The two sides are chords of the circle. A chord is a line segment whose endpoints also sit on the circumference of a circle. One endpoint is the vertex while the other endpoint sits across the circle. The arc formed by the inscribed angle is called the intercepted arc. This arc is part of the circumference of the circle that is between the two chords of the angle, or intercepted by the chords. The intercepted arc and the inscribed angle have a special relationship.

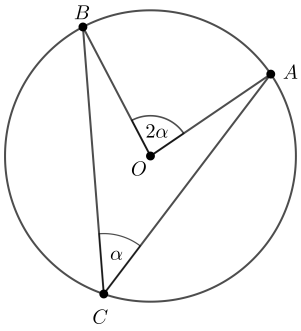


II. THEOREMS & PROPERTIES

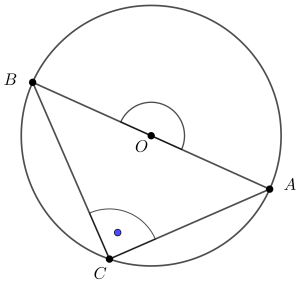
1. **Alternate Segment Theorem**. In any circle, the angle between a chord and a tangent through one of the endpoints of the chord is equal to the angle in the alternate segment, i.e. the angle subtended by the chord in the opposite side of the previous angle.



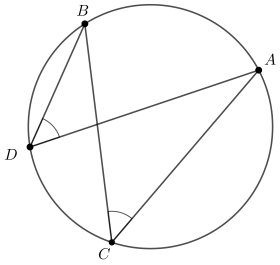
2. The Central Angle is twice the Inscribed Angle.



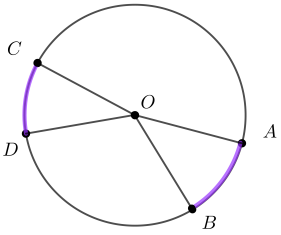
3. Angle inscribed in semicircle is 90°.



4. Inscribed angles subtended by the same arc are equal.



5. Central angles subtended by arcs of the same length are equal.



SOURCES:

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