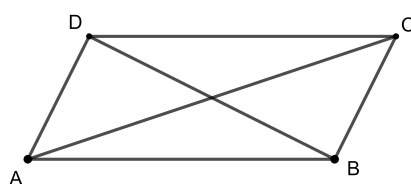# GEOMETRY ON THE PLANE (3)
## QUADRILATERALS
VOCABULARY, PROPERTIES & EXERCISES

1. CLASSIFICATION & PROPERTIES OF QUADRILATERALS

   Polygons with four sides are called **QUADRILATERALS**.

   (a) **A PARALLELOGRAM**. Opposite sides are parallel and congruent. Consecutive angles are supplementary. The diagonals bisect.


   (b) **A RHOMBUS**. It is a parallelogram with four congruent sides. In addition to all the properties of a parallelogram, the diagonals are perpendicular and bisect the opposite angles.
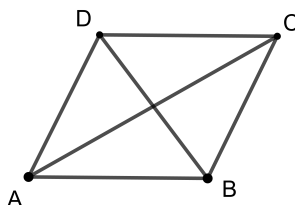

   (c) **A RECTANGLE**. It is a parallelogram with four right angles. In addition to all the properties of a parallelogram, the diagonals are congruent.
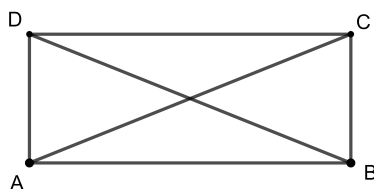

   (d) If a rectangle is also a rhombus, then it is called a **SQUARE**. A square would have all the properties listed previously.
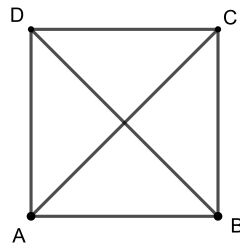
(e) **A TRAPEZOID**. It has exactly one set of parallel sides called bases. The segment connecting the midpoints of the non-parallel sides is parallel to the bases and has length equal to the average of the bases.
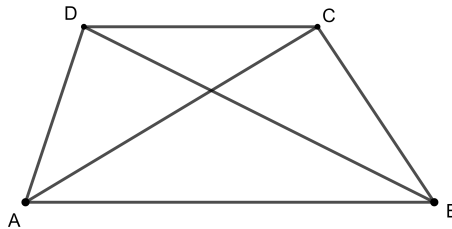
(f) **A KITE**. It has one pair of opposite angles congruent and the diagonals are perpendicular.
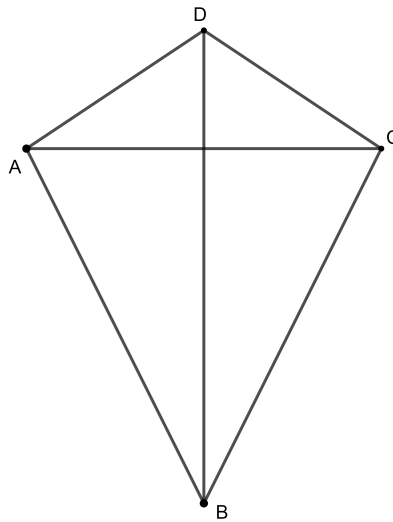
2. AREA OF QUADRITERALS

The area of a polygonal region can be computed as the sum of the areas of subtriangles. In what follows we consider some special cases with explicit formulas. They are easy consequences of the triangle decomposition. The area of a

- parallelogram is the product of one of the parallel bases and the altitude belonging to this base. The trigonometric version of the formula is

$$A = ab \sin \alpha$$

This follows easily from the division of the parallelogram into congruent triangles by one of the diagonals.

- trapezoid can be computed as

$$A = \frac{a+c}{2}m,$$

where a and c are the lengths of the parallel bases and m is the altitude of the trapezoid. One can introduce the mid-line segment for trapezoids on the model of triangles in the same way: the midline of a trapezoid is just the line segment joining the midpoints of the legs. Using the division of the trapezoid into triangles by one of the diagonals it can be easily seen that the length of the midline of a trapezoid is just the arithmetic mean of the lengths of the parallel bases. Another way to conclude the area formula is to put two congruent copies of the trapezoid next to each other in such a way that they form a parallelogram. In terms of geometric transformation it can be realized by a central reflection about the midpoint of one of the legs.

- convex quadrilateral is just

$$A = \frac{ef}{2} \sin \alpha,$$

where e and f are the lengths of the diagonals and α is the angle enclosed by them.

3. EXERCISES

Exercise 1 *Prove the area formula of a parallelogram.*

Exercise 2 *Prove the area formula of a trapeziod.*

Exercise 3 *Prove the area formula of a kite.*

- PARALLELOGRAM

    Exercise 4 *The shortest diagonal of a parallelogram has length 8, the angle of the diagonals is $45°$, and its area is 40. Calculate the perimeter of the parallelogram*

- RHOMBUS

    Exercise 5 *The perimeter of the rhombus is 40, its area is 96. What are the angles, sides, and diagonals of the rhombus.*

    Exercise 6 *The length of the side of a rhombus is just the geometric mean of the diagonals. What is the ratio of the two diagonals.*

- RECTANGLE

    Exercise 7 *The sides AB and BC of rectangle $ABCD$ are 10 and 6. What is the distance of a point P on the side AB from the vertex D if $|AP| + |PC| = 12$.*

    Exercise 8 *In rectangle $ABCD$ side AB is three times longer then BC. The distance of an interior point P from the vertices B, A and D is $|PB| = 4\sqrt{2}$, $|PA| = \sqrt{2}$ and $|PD| = 2$. What is the area of the rectangle.*

- SQUARE

  Exercise 9 *The side of the square ABCD is* 10. *Calculate the radius of the circle which passes through the point A, and touches the sides BC and CD.*

- TRAPEZOID

  Exercise 10 *Three sides of a symmetrical trapezoid are of length* 10. *The fourth side has length* 20. *Calculate the angles and the area of the trapezoid.*

  Exercise 11 *In a symmetrical trapezoid the inclination angle of the diagonal to the longer parallel base is* 45 *degree, the length of the diagonal is* 10. *What is the area of the trapezoid?*

  Exercise 12 *The length of the mid - line of a symmetric trapezium is* 10, *the diagonals are perpendicular to each other. What is the area of the trapezium.*

  Exercise 13 *The diagonals of a trapezium are perpendicular. The lengths of the parallel sides are* 17 *and* 34, *one of the legs is* $\sqrt{964}$. *How long is the second leg, what is the area, and the height of the trapezium.*

- KITE

  Exercise 14 *In kite ABCD we know that* $|AB| = |BC| = 2$ *and* $|CD| = |DA|$. *At vertex A the angle is* 120°, *and at D the angle is* 60°. *Calculate the unknown angles, sides and diagonals of the kite and furthermore, the radius of the inscribed circle.*

SOURCES:

1. C. Vincze & L.Kozma, 2014, 'College Geometry' http://math.unideb.hu/.

2. J. Roe, 1993, 'Elementary Geometry', Oxford University Press.

3. H. S. M. Coxeter, 1969, 'Introduction to Geometry', John Wiley and Sons, Inc., Second Edition, .