GEOMETRY ON THE PLANE (5)

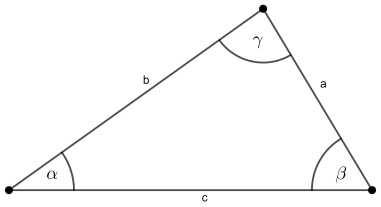
THE LAW OF SINES & THE LAW OF COSINES

THEOREMS & EXERCISES

1. THE LAW OF SINES

Theorem 1 (The Law of Sines) *If α, β, and γ are the measurements of the angles of an oblique triangle, and a, b, and c are the lengths of the sides opposite of the corresponding angles, then the ratios of the a side's length to the sine of the angle opposite the side must all be the same.*

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$



Picture 1

2. THE LAW OF COSINES

Theorem 2 (The Law of Cosines) *For any triangle (e.g. given in the Picture 1) the following equations are true:*

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

3. EXERCIES (APPLICATION PROBLEMS)

Exercise 1 *Plane A is flying directly toward the airport which is $20°$ miles away. The pilot notice a second plane, B, $45°$ to her right. Plane B is also flying directly towards the airport. The pilot of plane B calculates that plane A is $50°$ to his left. Based on that information how far is plane B from the airport? Give your answer to 3 significant figures.*

Exercise 2 *Hannah measures the angle of elevation of the peak if a mountain at $35°$. Steve who is 1200 feet closer to the mountain, along a straight path, measures the angle of elevation as $42°$. How high to the nearest foot is the mountain?*

Exercise 3 *Two ships, A and B, leave a port in directions that are $41°$ from each other. The speed of the slower ship, A is $7km/h$. After half an hour the captain can see Ship B on a bearing of $110°$ from Ship A. How far apart are the ships? Give your answer correct to one decimal place.*

Exercise 4 *A playground slide has a* 2.3 *metre ladder. The slide measures* 4.6 *metres. The ladder makes an angle of* 67° *with the ground. Calculate the angle the slide makes with the ground.*

Exercise 5 *Flights* 104 *and* 217 *are both approaching O'Hare International Airport fro directions directly opposite one another and at an altitude of* 2.5 *miles. The pilot on flight* 104 *reports an angle of depression of* 17° *to the tower, and the pilot on flight* 217 *reports an angle of depression of* 12° *to the tower. Calculate the distance between the planes.*

Exercise 6 *On a map, Orlando is* 178 *mm due south of Niagara Falls, Denver is* 273 *mm from Orlando, and Denver is* 235 *mm from Niagara Falls. Find the angle at Niagara Falls.*

Exercise 7 *Nicole shines a light from a window of a lighthouse on a cliff* 250 *feet above the water level. Nick,* 10 *feet above the water level in a ship off shore, finds that the angle ofelevation of the light is* 3°. *Find the length of the line of sight (light beam) from the ship to Nicole. Round to the nearest tenth.*

Exercise 8 *Fire towers A and B are located* 10 *miles apart. Rangers at fire tower A spot a fire at* 42°, *and rangers at fire tower B spot the same fire at* 64°. *How far from tower A is the fire to the nearest tenth of a mile?*