# EXPONENTIAL FUNCTIONS & LOGARITHMIC FUNCTIONS

THEORY & PROBLEMS

## I THEORY

### 1. EXPONENTIAL FUNCTION

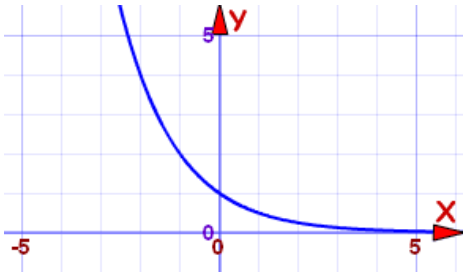
The general Exponential Function is given by formula:

$$f(x) = a^x$$

$a$ is a value greater than 0, not equal to 1.

Properties depend on value of $a$. When $a = 1$, the graph is a horizontal line at $y = 1$. Apart from that there are two cases to look at:

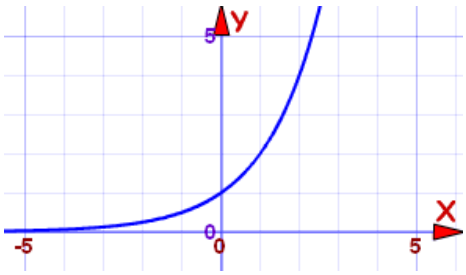
**CASE 1.** $a \in (0; 1)$




PROPERTIES:

- Domain – $D = \mathbf{R}$;
- Range – $ZW_f = (0; +\infty)$;
- As $x$ increases, $f(x)$ heads to 0;
- As $x$ decreases, $f(x)$ heads to infinity;
- it is a STRICTLY DECREASING function;
- it has a HORIZONTAL ASYMPTOTE along the $x$-axis ($y = 0$).
- It always intersects the $y$-axis at $y = 1$, in other words it passes through $(0, 1)$

**CASE 2.** $a \in (1; +\infty)$



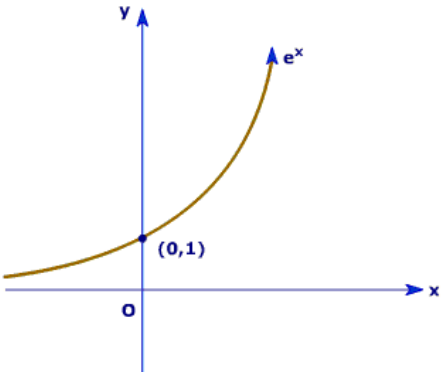

PROPERTIES:

- Domain – $D = \mathbf{R}$;
- Range – $ZW_f = (0; +\infty)$;
- As $x$ increases, $f(x)$ heads to infinity;
- As $x$ decreases, $f(x)$ heads to 0;
- it is a STRICTLY INCREASING function;
- it has a HORIZONTAL ASYMPTOTE along the $x$-axis ($y = 0$).
- It always intersects the $y$-axis at $y = 1$, in other words it passes through $(0, 1)$

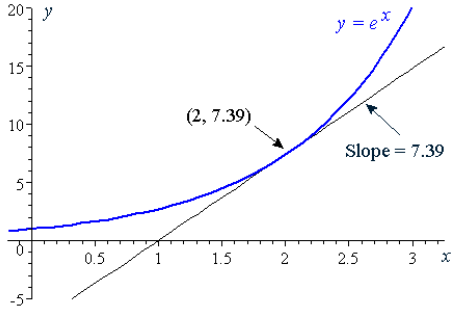
**SPECIAL CASE** $a = e \approx 2.718281828459...$ (natural exponential function)

$$f(x) = e^x$$

$e$ – – is known as Euler's Number.




At any point the slope of $e^x$ equals the value of $e^x$ :
when $x = 0$, the value of $e^x = 1$, and slope $= 1$
when $x = 1$, the value of $e^x = e$, and slope $= e$
etc...




4.png

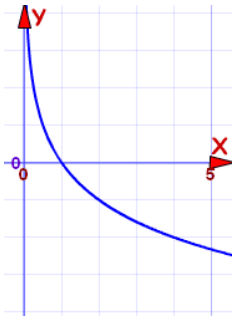
***half-life*** – the time it takes to decay to one-half of a given quantity.

2. LOGARITHMIC FUNCTION

The logarithmic function:

$$f(x) = \log_a(x)$$

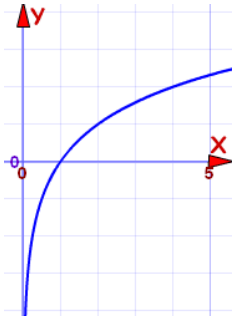
$a$ is any value greater than 0, except 1.




**CASE 1.** $a \in (0;1)$

- Domain – $D = (0;+\infty)$
- Range – $ZW_f = \mathbf{R}$
- Zero of the function $x = 1$
- As $x$ nears 0, it heads to infinity
- As $x$ increases it heads to minus infinity
- It is a Strictly Decreasing function
- It has a Vertical Asymptote along the $y$-axis ($x = 0$).

**CASE 2.** $a \in (1;+\infty)$




- Domain – $D = (0;+\infty)$
- Range – $ZW_f = \mathbf{R}$
- Zero of the function $x = 1$
- As $x$ nears 0, it heads to minus infinity
- As $x$ increases it heads to infinity
- It is a Strictly Increasing function
- It has a Vertical Asymptote along the $y$-axis ($x = 0$).

**SPECIAL CASE** $a = e \approx 2.718281828459...$ (natural log function)

$$f(x) = \ln(x)$$

it is the logarithmic function with base $e$.

3. PROBLEMS

(a) Sketch the graph of the function $f(x) = a^x$ for the following values of $a$, on the same axes.n

- $a = 3$,
- $a = 6$,
- $a = 1$,
- $a = \frac{1}{3}$
- $a = \frac{1}{6}$

(b) Sketch the graph of the function $f(x) = \log_a(x)$ for the following values of $a$, on the same axes.

- $a = 3$,
- $a = 6$,
- $a = 1$,
- $a = \frac{1}{3}$
- $a = \frac{1}{6}$

(c) Sketch the graph and determine the domain and range: $f(x) = 5^{-x} - 10$.

(d) Sketch the graph and determine the domain and range: $f(x) = 2^{x-1} + 3$.

(e) An investment of \$500 is made in a 6-year CD that earns $4\frac{1}{2}\%$ annual interest that is compounded monthly. How much will the CD be worth at the end of the 6-year term?
Answer: \$654.65

(f) Under optimal conditions *Escherichia coli (E. coli)* bacteria will grow exponentially with a doubling time of 20 minutes. If 1,000 E. coli cells are placed in a Petri dish and maintained under optimal conditions, how many E. coli cells will be present in 2 hours?
Answer: 64.000 cells

(g) Due to radioactive decay, caesium-137 has a half-life of 30 years. How long will it take a 50-milligram sample to decay to 10 milligrams?
Answer: $\approx 69.66 years$

FOR MORE EXERCISES AND PROBLEMS CLICK THE LINK BELOW:

1. https://2012books.lardbucket.org/pdfs/advanced-algebra/s10-exponential-and-logarithmic-fu.pdf