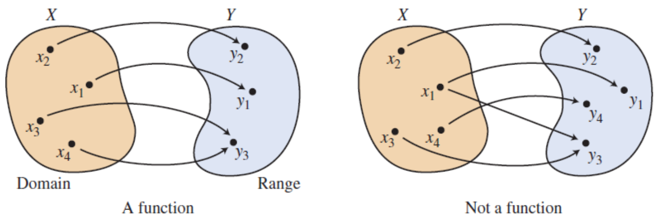
1.3 FUNCTIONS
THEORY & PROBLEMS

1. Function

Definition 1 ***A function*** *from a set D to a set R is a rule that assigns to every element in D a unique element in R . The set D of all* ***input values*** *is* ***the domain*** *of the function, and the set R of all* ***output values*** *is* ***the range*** *of the function.*

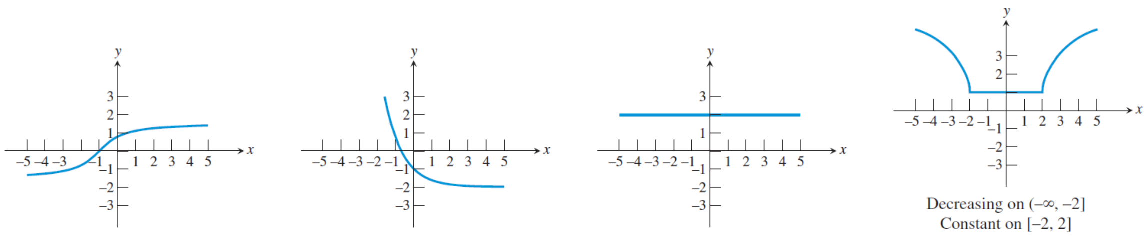
There are many ways to look at functions. One of the most intuitively helpful is the "`machine`" concept, in which values of the domain (x) are fed into the machine (the function f) to produce range values (y). To indicate that y comes from the function acting on x , we use **Euler's function notation** $y = f(x)$ (which we read as " y **equals** f **of** x " or " **the value of** f **at** x "). Here x is **the independent variable** and y is **the dependent variable** .



A function
(a)

Not a function
(b)

2. Increasing and Decreasing Functions



Increasing

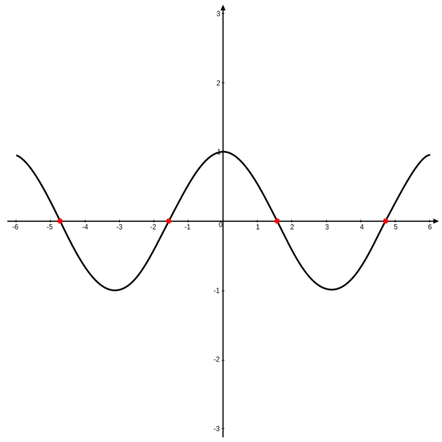
Decreasing

Constant

Decreasing on (−∞, −2]
Constant on [−2, 2]
Increasing on [2, ∞)

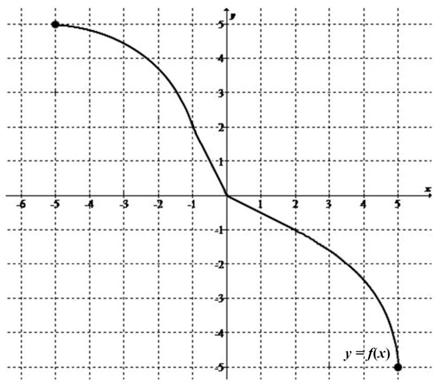
3. A Zero of a function.

Definition 2 *An input x is a zero of a function f if and only if $f(x) = 0$.*



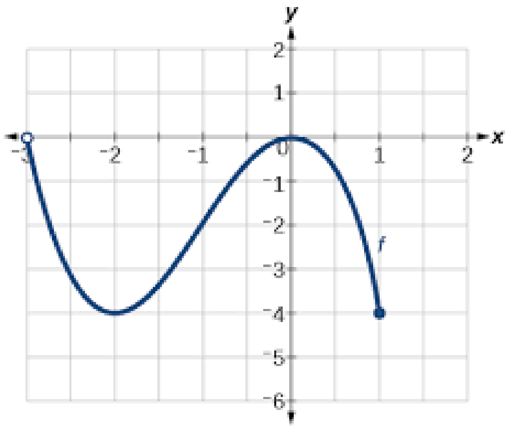
4. Problems

Problem 1. The illustration below represents a function f.



- The domain D_f of the function f is ....
- The range ZW_f of the function f is ....
- The zero of the function f is ....
- The function is .... on $[-5; 5]$.
- For the argument $x =$ .... the value $y = 2$

Problem 2. Fill the gaps.



- $f(-2) =$ $\qquad$ $f(-1) =$ $\qquad$ $f(-2.5) =$
- The domain of the function is the set (interval) D_f..........
- .The range of the function f is the set ZW_f ....
- The zeros of the function is input ....
- The output of the function f for the input $x = 0$ equals ....
- Positive outputs of the function f: $f(x) > 0$ for $x \in$ ....
- Negative outputs of the function f: $f(x) < 0$ for $x \in$ ....
- The maximum interval in which the function increases is: ....
- The maximum intervals in which the function decreases are: ....
- The function f takes the greatest value $y =$.................... at a point $x =$.............................
- The function f takes the smallest value $y =$.................... at a point $x =$.............................

Problem 3. Find the domain of the function:

(a) $f(x) = \frac{1}{1-x^2}$

(b) $f(x) = \frac{3-x}{x^2+2x-15}$

(c) $f(x) = \sqrt{-x^2 + x + 30}$

(d) $f(x) = \frac{1}{\sqrt{x-3}} + \sqrt{x+2}$

(e) $f(x) = \frac{\sqrt{|x|-7}}{|x+4|-3}$

Problem 4. Find the domain and zeros of the function:

(a) $f(x) = \frac{(x+3)(x-2)}{9-x^2}$

(b) $f(x) = \frac{3-4x}{x^2+2x-12}$

(c) $f(x) = \sqrt{-5x^2 + x + 6}$

(d) $f(x) = \sqrt{(x-7)(x+2)}$

(e) $f(x) = \frac{\sqrt{|x|-3}}{|x+2|-1}$

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