Problems

1.1. Give another description of the following sets

a. {x is a real number such that $x^2 = 1$ } (Ans:{1,-1})

b. {x: x is an integer such that $x^2 - 3 = 0$ } (Ans: Φ)

c. {x: x is an integer such that $x^2 - 3x + 2 = 0$ } (Ans: {1,2})

d. {1, 3, 5, 7, 9} (Ans: {x is positive integer ,x is odd and less than 10})

1.2. Consider the universal set $U = \{1, 2, ..., 9\}$ and sets A = {1, 2, 5, 6}, B = {2, 5, 7}, C = {1, 3, 5, 7, 9}. Find: (a) $A \cap B$ and $A \cap C$. (Ans:{2,5},{1,5}) (b) $A \cup B$ and $B \cup C$. (Ans: {1, 2, 5, 6, 7}, {1, 2, 3, 5, 7, 9}) (Ans:{3, 4, 7, 8, 9},{2, 4, 6, 8}) (c) A^c and C^c . (d) A - B and A - C (Ans:{1,6},{2,6}) (e) A Δ B and A Δ C (Ans:{1,6,7},{2,3,6,7,9}) $(f)(A \cup C) - B \text{ and } (B \Delta C) - A \quad (Ans: \{1,3,6,9\}, \{3,9\})$ **1.3.** Given that $A = \{2, 4\}$ and $B = \{x: x^2 + 6x + 8 = 0\}$ Are A and B disjoint sets? (Ans : yes) **1.4.** Which of the following sets are equal? $A = \{x \mid x^2 - 4x + 3 = 0\}, C = \{x \mid x \in \mathbf{N}, x < 3\}, E = \{1, 2\}, G = \{3, 1\},$ $B = \{x | x^2 - 3x + 2 = 0\}, D = \{x | x \in N, x \text{ is odd}, x < 5\}, F = \{1, 2, 1\}, H$ $= \{1, 1, 3\}.$ (Ans: A = D = G = H , B = C = E = F)

1.5. Let M, P and C be the sets of students taking Mathematics, Physics and Computer science respectively in a University. Assume 300 students study Mathematics,350 students study Physics,450 study Computer science ,100 study mathematics and Physics,150 study Mathematics and Computer science ,75 study Physics and Computer science and 10 study all three courses .How many students are taking exactly one of those courses? (Ans:480)

1.6. We form the union of a set with 5 elements and a set with 9 elements. Which of the following numbers can we get as the cardinality of the union: 4, 6, 9, 10, 14, 20 ? (Ans:9, 10, 14)

1.7. What is the intersection of

(a) the sets {0,1,3} and {1,2,3}? (Ans: {1, 3})

(b) the set of girls in this class and the set of boys in this class? (Ans: Φ)

(c) the set of prime numbers and the set of even numbers? (Ans:{2})

1.8. What is the symmetric difference of the set Z_+ of non-negative integers and the set E of even integers ($E = \{...-4, -2, 0, 2, 4, ...\}$ contains both negative and positive even integers). (Ans: The set of negative even integers and positive odd integers.)

1.9. Prove that if $A \subset B$ then i. $A \cap B = A$ ii. $A \cup B = B$

1.10. Determine whether each of the following is true or false:

$a. x \in \{x\}$	$b. \{x\} \subseteq \{x\}$	$c. \{x\} \in \{x\}$
d. $\{x\} \in \{\{x\}\}$	$e. \Phi \subseteq \{x\}$	$f.\Phi\in\{x\}$

(Ans:a. True b.True c. False d. True e. True f. False)

1.11. Let A and B are two sets prove that

$$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$$

1.12. Write the dual of each set equation

a. $(U \cap A) \cup (B \cap A) = A$. (Ans: $(\Phi \cup A) \cap (B \cup A) = A$

b. $(A \cap U) \cup (B \cap A) = A$. $(Ans: (A \cup \Phi) \cap (B \cup A) = A)$

1.13. Prove the following:

a. A \subseteq B if and only if A \cap B^c = Ø

b. A \subseteq B if and only if A^c \cup B = U

c. A \subseteq B if and only if A\B = Ø

1.14. Find the power set P(A) of $A = \{1, 3, 5\}$.

 $[\Phi,\{1\},\{3\},\{5\},\{1,3\},,\{1,5\},\{3,5\},\{1,3,5\}]$

1.15. Let A and B be arbitrary sets. Show that $(A - B) \cap B = \Phi$

1.16. Let A = $\{1, 2, 3, 4, 5, 6\}, A_1 = \{1, 2\}, A_2 = \{3, 4\}, A_3 = \{5, 6\}.$

Show that $\{A_1, A_2, A_3\}$ is a partition of A.

1.17. Determine whether or not each of the following is a partition of the set **N** of positive integers:

a. $[\{n \mid n > 5\}, \{n \mid n < 5\}]$. (Ans:No) b. $[\{n \mid n > 6\}, \{1, 3, 5\}, \{2, 4\}]$. (Ans:No) c . $[\{n \mid n^2 > 11\}, \{n \mid n^2 < 11\}]$. (Ans:Yes)