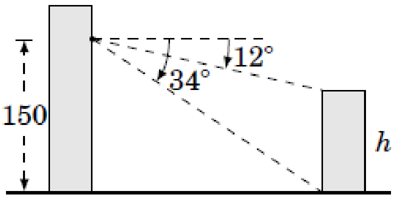
1.9. GEOMETRY ON THE PLANE (3) – TRIGONOMETRY – EXERCISES & PROBLEMS

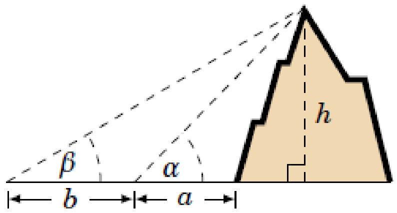
(based on Micheal Coral – "Trigonometry")

A. Trigonometry of the acute angle.

1. From a position 150 ft above the ground, an observer in a building measures angles of depression of 12° and 34° to the top and bottom, respectively, of a smaller building, as in the picture on the right. Use this to find the height h of the smaller building.

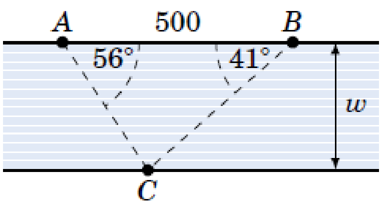


2. Generalize Example 1.12: A person standing a ft from the base of a mountain measures an angle of elevation α from the ground to the top of the mountain. The person then walks b ft straight back and measures an angle of elevation β to the top of the mountain, as in the picture on the right. Assuming the ground is level, find a formula for the height h of the mountain in terms of a, b, α, and β.



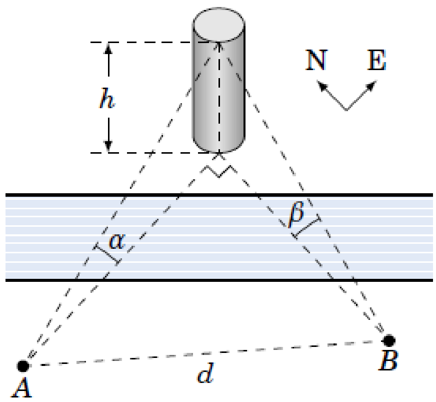
3. As the angle of elevation from the top of a tower to the sun decreases from 64° to 49° during the day, the length of the shadow of the tower increases by 92 ft along the ground. Assuming the ground is level, find the height of the tower.

4. Two banks of a river are parallel, and the distance between two points A and B along one bank is 500 ft. For a point C on the opposite bank, $\angle BAC = 56°$ and $\angle ABC = 41°$, as in the picture on the right. What is the width w of the river?
(*Hint: Divide* $\overline{AB}$ *into two pieces.*)



5. A tower on one side of a river is directly east and north of points A and B, respectively, on the other side of the river. The top of the tower has angles of elevation α and β from A and B, respectively, as in the picture on the right. Let d be the distance between A and B. Assuming that both sides of the river are at the same elevation, show that the height h of the tower is

$$h = \frac{d}{\sqrt{(\cot\alpha)^2 + (\cot\beta)^2}}.$$



B. Trigonometry of any angle

For Exercises 1-10, state in which quadrant or on which axis the given angle lies.

1. $127°$ **2.** $-127°$ **3.** $313°$ **4.** $-313°$ **5.** $-90°$

6. $621°$ **7.** $230°$ **8.** $2009°$ **9.** $1079°$ **10.** $-514°$

11. In which quadrant(s) do sine and cosine have the same sign?

12. In which quadrant(s) do sine and cosine have the opposite sign?

13. In which quadrant(s) do sine and tangent have the same sign?

14. In which quadrant(s) do sine and tangent have the opposite sign?

15. In which quadrant(s) do cosine and tangent have the same sign?

16. In which quadrant(s) do cosine and tangent have the opposite sign?

For Exercises 17-21, find the reference angle for the given angle.

17. $317°$ **18.** $63°$ **19.** $-126°$ **20.** $696°$ **21.** $275°$

For Exercises 22-26, find the exact values of $\sin\theta$ and $\tan\theta$ when $\cos\theta$ has the indicated value.

22. $\cos\theta = \frac{1}{2}$ **23.** $\cos\theta = -\frac{1}{2}$ **24.** $\cos\theta = 0$ **25.** $\cos\theta = \frac{2}{5}$ **26.** $\cos\theta = 1$

For Exercises 27-31, find the exact values of $\cos\theta$ and $\tan\theta$ when $\sin\theta$ has the indicated value.

27. $\sin\theta = \frac{1}{2}$ **28.** $\sin\theta = -\frac{1}{2}$ **29.** $\sin\theta = 0$ **30.** $\sin\theta = -\frac{2}{3}$ **31.** $\sin\theta = 1$

For Exercises 32-36, find the exact values of $\sin\theta$ and $\cos\theta$ when $\tan\theta$ has the indicated value.

32. $\tan\theta = \frac{1}{2}$ **33.** $\tan\theta = -\frac{1}{2}$ **34.** $\tan\theta = 0$ **35.** $\tan\theta = \frac{5}{12}$ **36.** $\tan\theta = 1$

C. Trigonometric Identities

For Exercises 2 and 3, find the exact values of $\sin(A+B)$, $\cos(A+B)$, and $\tan(A+B)$.

2. $\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\sin B = \frac{24}{25}$, $\cos B = \frac{7}{25}$ **3.** $\sin A = \frac{40}{41}$, $\cos A = \frac{9}{41}$, $\sin B = \frac{20}{29}$, $\cos B = \frac{21}{29}$

4. Use $75° = 45° + 30°$ to find the exact value of $\sin 75°$.

5. Use $15° = 45° - 30°$ to find the exact value of $\tan 15°$.

6. Prove the identity $\sin\theta + \cos\theta = \sqrt{2}\,\sin(\theta + 45°)$. Explain why this shows that

$$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$$

for all angles θ. For which θ between $0°$ and $360°$ would $\sin\theta + \cos\theta$ be the largest?

For Exercises 7-14, prove the given identity.

7. $\cos(A+B+C) = \cos A\,\cos B\,\cos C - \cos A\,\sin B\,\sin C - \sin A\,\cos B\,\sin C - \sin A\,\sin B\,\cos C$

8. $\tan(A+B+C) = \dfrac{\tan A + \tan B + \tan C - \tan A\,\tan B\,\tan C}{1 - \tan B\,\tan C - \tan A\,\tan C - \tan A\,\tan B}$

9. $\cot(A+B) = \dfrac{\cot A\,\cot B - 1}{\cot A + \cot B}$ **10.** $\cot(A-B) = \dfrac{\cot A\,\cot B + 1}{\cot B - \cot A}$

ANSWERS:

A:

1. 102.7 ft **3.** 241.1 ft **4.** 274 ft

B:

1. QII **3.** QIV **5.** negative y-axis **7.** QIII **9.** QIV **11.** QI, QIII **13.** QI, QIV **15.** QI, QII **17.** 43° **19.** 54° **21.** 85° **23.** $\sin\theta = \sqrt{3}/2$ and $\tan\theta = -\sqrt{3}$; $\sin\theta = -\sqrt{3}/2$ and $\tan\theta = \sqrt{3}$
25. $\sin\theta = \sqrt{21}/5$ and $\tan\theta = \sqrt{21}/2$;

$\sin\theta = -\sqrt{21}/5$ and $\tan\theta = -\sqrt{21}/2$
27. $\cos\theta = \sqrt{3}/2$ and $\tan\theta = 1/\sqrt{3}$; $\cos\theta = -\sqrt{3}/2$ and $\tan\theta = -1/\sqrt{3}$
29. $\cos\theta = \pm 1$ and $\tan\theta = 0$
31. $\cos\theta = 0$ and $\tan\theta$ is undefined
33. $\sin\theta = 1/\sqrt{5}$ and $\cos\theta = -2/\sqrt{5}$; $\sin\theta = -1/\sqrt{5}$ and $\cos\theta = 2/\sqrt{5}$
35. $\sin\theta = 5/13$ and $\cos\theta = 12/13$; $\sin\theta = -5/13$ and $\cos\theta = -12/13$

C:

3. $\sin(A+B) = \frac{1020}{1189}$, $\cos(A+B) = -\frac{611}{1189}$, $\tan(A+B) = -\frac{1020}{611}$ **4.** $(\sqrt{6}+\sqrt{2})/4$
5. $2-\sqrt{3}$ **15.** Hint: For $a \neq 0$ and $b \neq 0$, draw a right triangle with legs of lengths a and b.